18. Solve:

$u_{tt} + a^2 u_{xxxx} = 0,\ 0 < x < l,\ t > 0$

$u(x,0) = f(x),\ 0 \le x \le l$

$u_t(x,0) = g(x),\ 0 \le x \le l$

$u(0,t) = u(l,t) = 0,\ t > 0$

$u_{xx}(0,t) = u_{xx}(l,t) = 0,\ t > 0$

19. Obtain the solution of Dirichlet problem for a rectangle.

20. Explain the method of eigen functions to obtain the Green's function.

NOVEMBER/DECEMBER 2024

23PMA23 — PARTIAL DIFFERENTIAL EQUATIONS

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Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer all questions.

1. Classify the partial differential equation $y^2 u_{xx} - x^2 u_{yy} = 0$.

2. Write the Euler's equation.

3. Define Initial value problem.

4. Define adjoint operator.

5. Define Eigen function.

6. State uniqueness theorem.

7. Define Dirichlet problem.

8. State the minimum principle.

9. Define Green's function.

10. Write any two properties of Green's function.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Explain the conduction of heat in solids.

Or

(b) Obtain the canonical form of the equation $u_{xx} + x^2 u_{yy} = 0$.

12. (a) Derive d'Alembert solution of the Cauchy problem for one-dimensional wave equation.

Or

(b) Determine the solution of the following problem.

$u_{tt} = c^2 u_{xx},\ 0 < x < l, t > 0$

$u(x,0) = \sin\left(\frac{\pi x}{l}\right),\ 0 \le x \le l$

$u_t(x,0) = 0,\ 0 \le x \le 1$

$u(0,t) = 0,\ u(l,t) = 0,\ t > 0$

13. (a) Let $u(x,t)$ be a continuously differentiable function. If $u(x,t)$ satisfies the differential equation $u_t = ku_{xx}, 0 \le x \le l, t > 0$ and the initial conditions $u(x,0) = f(x),\ 0 \le x \le l$ and the boundary conditions $u(0,t) = 0$. $u(l,t) = 0, t > 0$ then prove that the solution is unique.

Or

(b) Obtain the solution of one dimensional wave equation using separation of variable method.

14. (a) State and prove maximum principle.

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Or

(b) Derive the solution of Neumann problem for a circle.

15. (a) Obtain the green's function solution of the Dirichlet problem involving the Helmholtz operator namely $\nabla^2 u + k^2 u = h$ in D, $u = f$ on B. Where D is a circular domain of unit radius with boundary B.

Or

(b) Solve $\nabla^2 u = h$ in $\eta > 0$, $u = f$ on $\eta = 0$ using the method of images.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Determine the general solution of

$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$

17. Obtain the solution of spherical wave equation.